


Greatest Integer Function

$$f(x) = a [b(x-h)] + k$$

[x] – Represents the greatest integer of x


$[x]=x$ where $x \in \mathbb{Z}$... round the number inside the [] down to the nearest integer

a – Vertical stretch or contraction

 **step height** – it is the distance between steps

If a is positive (+) steps go up ↗ if a is negative (-) steps go down ↘

b – Horizontal stretch or contraction

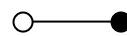
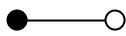
 **step length** – how long the step is

The step length is the reciprocal of the “**b**” term and it’s always positive $\frac{1}{|b|}$

Example: $f(x)=3[2(x-5)]-1$

$$b=2, \text{ therefore the step length is } \frac{1}{|b|} = \frac{1}{|2|} = \frac{1}{2}$$

If b is positive (+) steps go up ↗ if b is negative (-) steps go down ↘



h – Horizontal translation

k – Vertical translation

(h,k) – this is the coordinate of a “closed end point” (solid dot)

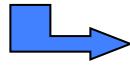
Greatest Integer Function

Properties

Domain – IR

Range – $k + an$, where $n \in \mathbb{Z}$

Variation – The graph is either increasing or decreasing

 increasing if $ab > 0$ (positive) ↗

decreasing if $ab < 0$ (negative) ↘

Sign – The graph is positive when it is above the x-axis

The graph is negative when it is below the x-axis

Zeros – The value of x , when $f(x) = 0$

There is either 1 zero or none...the zero is an interval

$$f(x) = 2 \lfloor -3(x - 5) \rfloor + 2$$

$$0 = 2 \lfloor -3(x - 5) \rfloor + 2 \quad \text{subtract 2 from each side}$$

$$-2 = 2 \lfloor -3(x - 5) \rfloor \quad \text{divide both sides by 2}$$

$$-1 = \lfloor -3(x - 5) \rfloor$$

is the number on the left an integer?

Yes! Then there is a zero and you can continue and solve for x . If No ☹, then there are no zeros.

$$-1 = -3(x - 5)$$

divide both sides by -3

$$\frac{1}{3} = x - 5$$

add 5 to both sides

$$\frac{16}{3} = x$$

y-intercept – The value of y , when $x = 0$

$$f(x) = 2 \lfloor -\frac{1}{3}(x - 5) \rfloor + 2$$

$$f(x) = 2 \lfloor -\frac{1}{3}(0 - 5) \rfloor + 2$$

$$f(x) = 2 \lfloor -\frac{1}{3}(-5) \rfloor + 2$$

$$f(x) = 2 \lfloor \frac{5}{3} \rfloor + 2$$

any fractions that appear within $\lfloor \rfloor$, are rounded down to the nearest integer.

$$f(x) = 2(1) + 2$$

$$f(x) = 4$$

Extrema – There is no maximum or minimum for the function

Greatest Integer Function

From Rule → Graph

- Need h and k – (h,k)
This is your first solid end point
- Is b positive or negative?
If $b > 0$, then the step is ●—○
If $b < 0$, then the step is ○—●
- What is the step length?
 $b=2$, therefore the step length is $\frac{1}{|b|} = \frac{1}{|2|} = \frac{1}{2}$
- What is the distance between the steps (height)?
 $a=?$
- Is the graph increasing or decreasing?
Increasing ($ab > 0$) – the steps are going up ↗
Decreasing ($ab < 0$) – the steps are going down ↘

With all this information you are ready to graph!

Greatest Integer Function

From Graph → Rule

Need:

- (h,k) – This can be any solid endpoint found on the graph
- a – The distance (height) between the steps
- b – The length of the step...remember $\frac{1}{|b|}$
- You can determine the sign of \underline{b} based on the direction of the step

If ● — ○ Then b is positive

If ○ — ● Then b is negative

- You can determine the sign of \underline{a} based on whether the graph is increasing or decreasing and depending on the sign of \underline{b} that you found above

If the graph is increasing then $ab > 0$

If \underline{b} is positive, then \underline{a} must be positive (+ • + = +)

If \underline{b} is negative, then \underline{a} must be negative (- • - = +)

If the graph is decreasing then $ab < 0$

If \underline{b} is positive, then \underline{a} must be negative (+ • - = -)

If \underline{b} is negative, then \underline{a} must be positive (- • + = -)

This is all the information you need to write your rule!